Existence and Stability of Dark Solitons in Bose-Einstein Condensate in Parabolic Trapped Optical Lattice

Vijendra Kumar*, O.P. Swami, A.K. Nagar

Abstract— In this paper, we consider dark solitons in one-dimensional Bose-Einstein condensate in parabolic trapped optical lattice. Analytical and numerical calculations are performed to determine the existence and stability of dark solitons. Our analysis is based on continuous Gross-Pitaevskii equation and discrete nonlinear Schrodinger equation. We show that the strength of external magnetic trap can change the stability of dark solitons. Stability windows of dark solitons are presented and stability approximations are derived using perturbation theory, with numerical results.

Index Terms—Bose-Einstein condensate, Dark solitons, Discrete nonlinear Schrodinger equation, External magnetic trap, Gross-Pitaevskii equation, Optical lattice, Perturbation theory.

1 Introduction

BOSE-Einstein Condensate (BEC) is a quantum physical phenomenon which occurs in many substances at very low temperatures [1]. In order to cool a system to observe condensation it is typical to trap the condensate with a potential. The governing equation for BEC needs to incorporate the nonlinear interactions of its constituents [2]. Thus the Discrete Nonlinear Schrodinger (DNLS) equation needs to be applied. The DNLS equation admits bright and dark solitons with focusing and defocusing nonlinearities, respectively. The dynamics of discrete dark solitons in presence of external magnetic trap with thermal and dynamical instabilities have been studied [3], [4], and [5]. Using Gross-Pitaevskii (GP) equation BEC trapped optical lattices have been described [6], [7], [8], [9], [10], and [11]. A controlled method for creating dark solitons by counter propagation of laser beams has been recently examined [12]. The long Bosonic Josephson junctions and BEC trapped optical lattices are also studied [13], [14].

In this paper we study dark solitons in BEC in parabolic trapped optical lattice. We use continuous GP equation and DNLS equation. We assume a parabolic shaped BEC, which can be described by one dimensional GP equation-

$$i\Psi_t = \Psi_{xx} + |\Psi|^2\Psi + [k(x^2+2x)] + V_0 \sin^2(\frac{2\pi x}{\lambda})\Psi \qquad (1)$$

here $\Psi(x,t)$ is the mean field wave function, k & V_0 are strength of external magnetic potential and optical lattice potential respectively, while λ is the wavelength of interference pattern created by counter propagated laser beams.

In tight binding limit, equation (1) becomes following DNLS equation

$$i\dot{\Psi} = -\varepsilon(\Psi_{n+1} + \Psi_{n-1} - 2\Psi_n) + |\Psi_n|^2\Psi_n + k(n^2+2n)\Psi_n \qquad (2)$$

where the over dot denotes the time derivative, ε is the coupling constant between two adjacent sites while n is the lattice site index.

In this paper we examine the condition for existence and stability of onsight dark solitons in DNLS. For small ε, the perturbation theory is used followed by numerical computations in MATLAB.

2 Analytical Set Up and Eigenvalue Equations

Stationary solution of system (2) are sought for in the form $\Psi_n = Z_n \exp(-i\mu t)$, where Z_n is a time independent and real valued wave function and it satisfies the stationary equation

$$-\varepsilon(Z_{n+1} - 2Z_n + Z_{n-1}) + |Z_n|^2 Z_n + k(n^2+2n)Z_n - \mu Z_n = 0 \qquad (3)$$

To examine the linear stability of Z_n, we introduce following ansatz

$\Psi_n = Z_n + \delta Y_n$ where δ<<1, and substitute this ansatz into equation (2), we find following linearized equation at $O(\delta)$:

$$i\dot{Y}_n = -\varepsilon(Z_{n+1} - 2Z_n + Z_{n-1})Y_n + 2|Z_n|^2 Y_n + X_n^2 Z_n + k(n^2+2n)Y_n - \mu Y_n = 0 \qquad (4)$$

writing $Y_n = A_n + iB_n$, and linearizing in δ, we find

$$\begin{pmatrix} \dot{A}_n \\ \dot{B}_n \end{pmatrix} = M \begin{pmatrix} A_n \\ B_n \end{pmatrix} \qquad (5)$$

where

$$M = \begin{pmatrix} 0 & L_+(\varepsilon) \\ -L_-(\varepsilon) & 0 \end{pmatrix} \qquad (6)$$

And the operators $L_+(\varepsilon)$ and $L_-(\varepsilon)$ are defined by

$$L_+(\varepsilon) = -\varepsilon(Z_{n+1} - 2Z_n + Z_{n-1}) + (Z_n^2 + k(n^2+2n) + \mu)$$

and

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$$L\_(\varepsilon) = -\varepsilon(Z_{n+1} - 2Z_n + Z_{n-1}) + (3Z_n^2 + k(n^2 + 2n) - \mu)$$

Let the eigenvalues of M be denoted by id, which implies that Z_n is stable if $\mathrm{Im}(d) = 0$. Since equation (6) is linear, we can eliminate one of the 'eigenvectors', for instance B_n, and then we obtain the following eigenvalue problems

$$L_+(\varepsilon)L_-(\varepsilon) = \lambda^2 A_n = \omega A_n \quad (7)$$

3 ANALYTICAL CALCULATIONS

In the uncoupled limit $\varepsilon = 0$ we denote the exact solutions of equation (3) by $Z_n = Z_n^{(0)}$, in which each Z_n must take one of the three values given by

$$0, \pm\sqrt{\mu - k(n^2 + 2n)}$$

Following Ref. [1], using a perturbative expansion, the dark soliton solutions are obtained as

$$Z_n = \begin{cases} \sqrt{\mu - k(n^2+2n)} - \dfrac{1}{2}\dfrac{\varepsilon}{\sqrt{\mu - k(n^2+2n)}} + O(\varepsilon^2), & n = 0,1 \\ \dfrac{\varepsilon}{\sqrt{\mu - k(n^2+2n)}} + O(\varepsilon^2), & n = -1,2 \\ O(\varepsilon^2) & \text{otherwise} \end{cases} \quad (8)$$

and its eigenvalues for small ε are given by

$$\omega = \mu^2 - k^2(n^2 + 2n)^2 - 4\mu\varepsilon + O(\varepsilon^2) \quad (9)$$

The instability of onsight discrete dark soliton is due to the collision of the smallest eigenvalue (9) with an eigenvalue bifurcating from lower and upper edge of continuous spectrum for small and large μ, respectively. Equating these quantities we find the critical value of μ as a function of coupling constant ε i.e.

$$k_{cr}^1 = \frac{1}{(n^2+2n)}\left[\frac{2}{5}\mu + \frac{8}{5}\varepsilon - \frac{1}{5}\sqrt{9\mu^2 - 28\mu\varepsilon - 16\varepsilon^2}\right] \quad (10)$$

$$k_{cr}^1 = \frac{1}{(n^2+2n)}[\mu^2 - 4\mu\varepsilon] \quad (11)$$

both k_{cr}^1 & k_{cr}^2 give approximation boundaries of the instability region in the (ε, k) plane.

4 COMPARISION WITH NUMERICAL CALCULATIONS AND DISCUSSION

Using Newton–Raphson method, we have numerically solved the static equation (2), and analyzed the stability of the numerical solution by solving the eigenvalue problem (4). We consider $\mu = 10$ in the model.

Figure 1 provides a full description of the dynamics of the parametrically driven DNLS model regarding the intervals of stability/instability of the model. Analytical prediction for the stability range as obtained by the conditions of collision of the phase mode eigenfrequency with the continuous spectrum from Equations (10) & (11).

Figure 2(a), 2(b) & 2(c) illustrate the typical stability/ instability scenario for different values of k (0.5, 0.2 & 0.1), with a fixed coupling constant $\varepsilon = 0.1$.

Figure 3(a), 3(b) & 3(c) illustrate the typical stability/ instability scenario for different values of ε (0.1, 0.6 & 2.0), with a fixed value of external magnetic potential k=0.1. In both cases we find that dark soliton is stable with a fixed range of ε and k, such values of ε and k are optimum. Ultimately the system governs oscillatory instability.

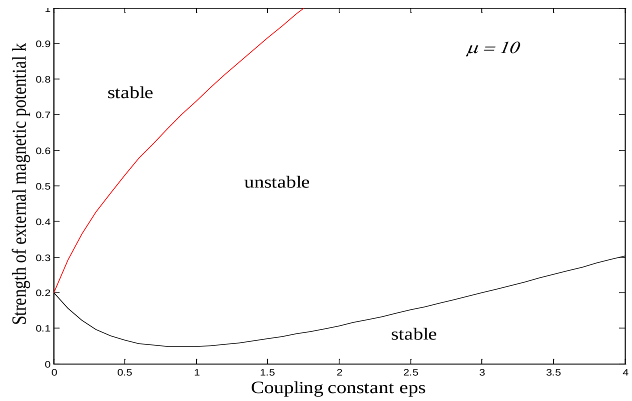

Fig. 1. The stability –instability region in the two parameter space $k - \varepsilon$. The solid red and black lines are the analytical approximations of equation (10) and (11).

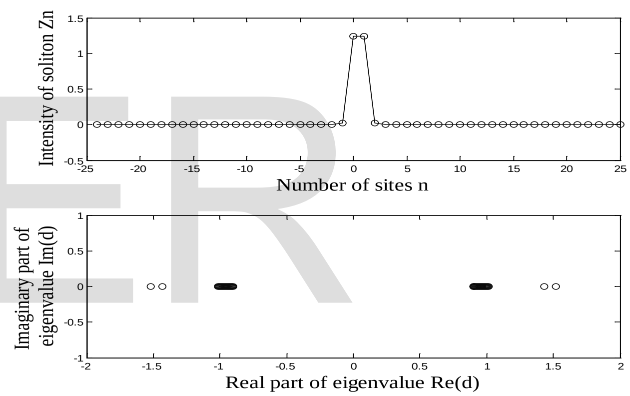

Fig. 2(a). The eigenvalue structure of intersite dark soliton for $\varepsilon = 0.1 \& k = 0.5$

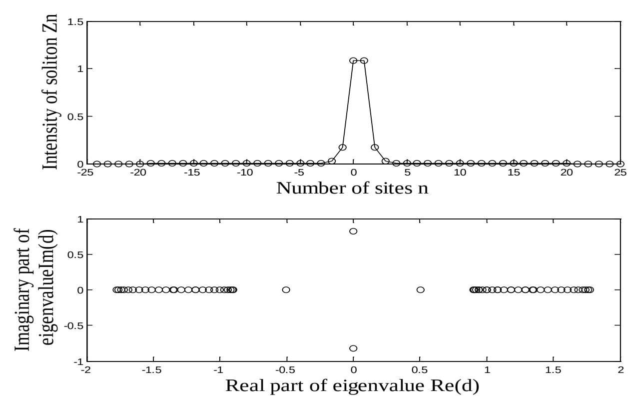

Fig. 2(b). The eigenvalue structure of intersite dark soliton for $\varepsilon = 0.1 \& k = 0.2$

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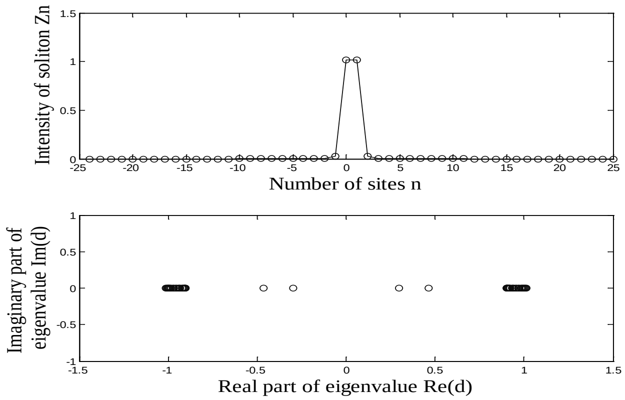

Fig. 2(c). The eigenvalue structure of intersite dark soliton for $\varepsilon = 0.1 \& k = 0.1$

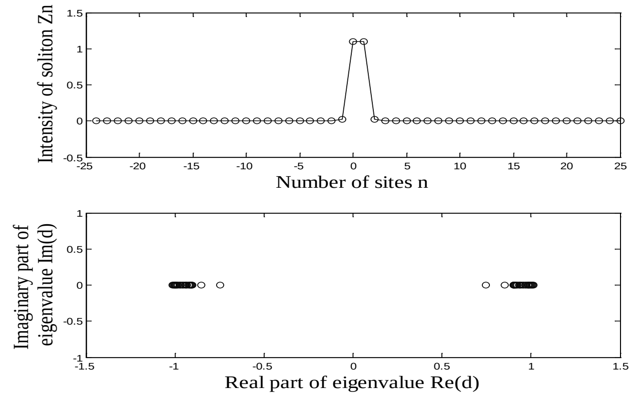

Fig. 3(c). The eigenvalue structure of intersite dark soliton for $k = 0.1 \& \varepsilon = 2$

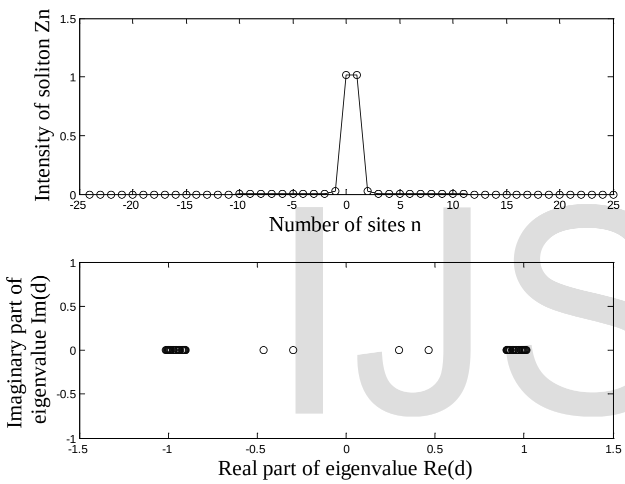

Fig. 3(a). The eigenvalue structure of intersite dark soliton for $k = 0.1 \& \varepsilon = 0.1$

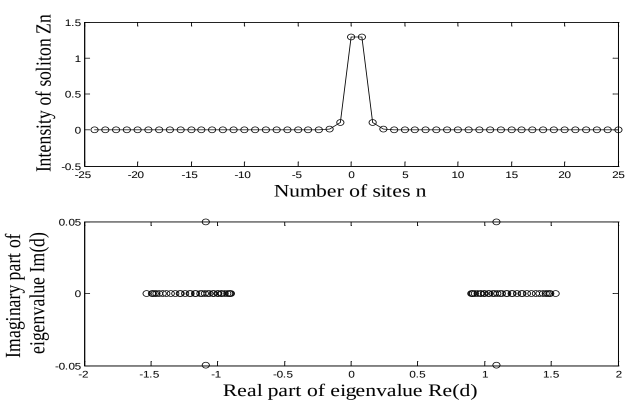

Fig. 3(b). The eigenvalue structure of intersite dark soliton for $k = 0.1 \& \varepsilon = 0.6$

5 CONCLUSION

In this paper, we have considered BEC in parabolic trapped optical lattice to determine the existence and stability of on-sight dark solitons.

Our analysis is based on continuous GP equation and DNLS equation. We have shown that the strength of external magnetic trap changed the stability of dark soliton. The existence and stability of onsight dark soliton is determined using perturbative analysis, which is followed by numerical computations in MATLAB.

We have considered the chemical potential as μ=10 and lattice index n=50. The result is an oscillatory instability which is expected from analysis.

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